ON $\mathcal{K}$-SETS AND NANO $\mathcal{O}$-SETS IN IDEAL NANO TOPOLOGICAL SPACES

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Abstract. we introduce the concepts of an ideal nano $\mathcal{K}$-set, an ideal nano $\mathcal{O}$-set, an ideal nano $\mathcal{K}_\alpha$-set and an ideal nano $\mathcal{O}_\alpha$-set are investigate and deal with an ideal nano topological spaces.

1. Introduction

An ideal I [7] on a space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [1]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [8] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the

2010 *Mathematics Subject Classification.* 54A05, 54A10, 54C05, 54C08, 54C10.
Key words and phrases. an ideal nano $\mathcal{K}$-set, an ideal nano $\mathcal{O}$-set an ideal nano $\mathcal{K}_\alpha$-set and an ideal nano $\mathcal{O}_\alpha$-set .

https://www.jartms.org/ ISSN: 2582-3078

$\star$-topology which is finer then τ. We will simply write $A^\star$ for $A^\star(I,\tau)$ and $\tau^\star$ for $\tau^\star(I,\tau)$. If I is an ideal on X, then (X,τ,I) is called an ideal topological space or an ideal space.

Parimala et al. [4, 3] introduced a new notions of ideal nano topological spaces.

In this chapter, we introduce the concepts of an ideal nano $\mathcal{K}$-set, an ideal nano $\mathcal{O}$-set, an ideal nano $\mathcal{K}_\alpha$-set and an ideal nano $\mathcal{O}_\alpha$-set are investigate and deal with an ideal nano topological spaces.

2. Preliminaries

Definition 2.1. [5] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

(3) *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. [2] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:*

(1) *U and $\phi \in \tau_R(X)$,*

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(2) *The union of the elements of any sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$,

(3) *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

Thus $\tau_R(X)$ *is a topology on* U *called the nano topology with respect to X and* $(U, \tau_R(X))$ *is called the nano topological space. The elements of* $\tau_R(X)$ *are called nano-open sets (briefly n-open sets). The complement of a nano open set is called nano closed.*

Throughout thesis, we denote a nano topological space by NTS. The nano-interior and nano-closure of a subset C *of* U *are denoted by* $Nint(C)$ *and* $Ncl(C)$, *respectively.*

A nano topological space NTS with an ideal I on U is called [4] an ideal nano topological space and is denoted by INTS. $H_n(x) = \{H_n \, |x \in H_n, H_n \in \tau_R(X)\}$, denotes [4] the family of nano open sets containing x.

Definition 2.3. [4] *Let INTS be a space with an ideal* I *on* U. *Let* $(.)^\star_n$ *be a set operator from* $\wp(U)$ *to* $\wp(U)$ *(*$\wp(U)$ *is the set of all subsets of* U*). For a subset* $C \subseteq U$, $A^{\star N}(I, \tau_R(X)) = \{x \in U : H_n \cap C \notin I,$ *for every* $G_n \in G_n(x)\}$ *is called the nano local function (briefly, nano local function) of C with respect to* I *and* $\tau_R(X)$. *We will simply write* $C^{\star N}$ *for* $C^{\star N}(I, \tau_R(X))$.

Theorem 2.4. [4] *Let INTS be a space and* A *and* B *be subsets of* U. *Then*

(1) $C \subseteq B \Rightarrow C^{\star N} \subseteq B^{\star N}$,

(2) $C^{\star N} = Ncl(C^{\star N}) \subseteq Ncl(C)$ *(*$C^{\star N}$ *is a nano closed subset of* $Ncl(C)$*)*,

(3) $(C^{\star N})^{\star N} \subseteq C^{\star N}$,

(4) $(C \cup B)^{\star N} = C^{\star N} \cup B^{\star N}$,

(5) $V \in \tau_R(X) \Rightarrow V \cap C^{\star N} = V \cap (V \cap C)^{\star N} \subseteq (V \cap C)^{\star N}$,

(6) $J \in I \Rightarrow (C \cup J)^{\star N} = C^{\star N} = (C - J)^{\star N}$.

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Theorem 2.5. [4] *Let INTS be a space with an ideal I and $C \subseteq C^{\star N}$, then $C^{\star N} = Ncl(C^{\star N}) = Ncl(C)$.*

Definition 2.6. [4] *Let an INTS. The set operator $Ncl^{\star}$ called a nano $\star$-closure is defined by $Ncl^{\star}(C) = C \cup C^{\star N}$ for $C \subseteq X$.*

It can be easily observed that $Ncl^{\star}(C) \subseteq Ncl(C)$.

Theorem 2.7. [3] *In a space INTS, if C and B are subsets of U, then the following results are true for the set operator $Ncl^{\star}$.*

(1) $C \subseteq Ncl^{\star}(C)$,
(2) $Ncl^{\star}(\phi) = \phi$ and $Ncl^{\star}(U) = U$,
(3) $IfC \subset B$, then $Ncl^{\star}(C) \subseteq Ncl^{\star}(B)$,
(4) $Ncl^{\star}(C) \cup Ncl^{\star}(B) = Ncl^{\star}(C \cup B)$.
(5) $Ncl^{\star}(Ncl^{\star}(C)) = Ncl^{\star}(C)$.

Definition 2.8. [6] *A subset C of space INTS is said to be*

(1) *an ideal nano α-open if $C \subset Nint(Ncl^{\star}(Nint(C)))$,*
(2) *an ideal nano pre-open if $C \subset Nint(Ncl^{\star}(C))$.*

3. On $\mathcal{K}$-sets and nano $\mathcal{O}$-sets in ideal nano spaces

Definition 3.1. *A subset C of a space INTS is called*

(1) *an ideal nano $\mathcal{K}$-set if $Nint(C) = Nint(Ncl^{\star}(C))$,*
(2) *an ideal nano $\mathcal{O}$-set if $C = Q \cap P$, where Q is nano open and P is an ideal nano $\mathcal{K}$-set,*
(3) *an ideal nano $\mathcal{K}_{\alpha}$-set if $Nint(C) = Nint(Ncl^{\star}(Nint(C)))$,*
(4) *an ideal nano $\mathcal{O}_{\alpha}$-set if $C = Q \cap P$, where Q is nano open and P is an ideal nano $\mathcal{K}_{\alpha}$-set.*

Example 3.2. *Let $U = \{1,2,3,4\}$ with $U/R = \{\{2\},\{4\},\{1,3\}\}$ and $X = \{3,4\}$. Then $\tau_R(X) = \{\phi, U, \{4\}, \{1,3\}, \{1,3,4\}\}$ and $I = \{\phi, \{3\}\}$.*

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(1) *the set* $\{2\}$ *is an ideal nano* $\mathcal{K}$*-set.*

(2) *the set* $\{4\}$ *is ideal nano* $\mathcal{K}_\alpha$*-set.*

(3) *the set* $\{2,3\}$ *is ideal nano* $\mathcal{O}$*-set*

(4) *the set* $\{1,3\}$ *is an ideal nano* $\mathcal{O}_\alpha$*-set*

Remark 3.3. *In a INTS,*

(1) *each nano open set is an ideal nano* $\mathcal{O}$*-set.*

(2) *each an ideal nano* $\mathcal{K}$*-set is an ideal nano* $\mathcal{O}$*-set.*

Remark 3.4. *The converses of Remark 3.3 are not true as shown in the next Examples.*

Example 3.5. *In Example 3.2,*

(1) *the set* $\{2\}$ *is an ideal nano* $\mathcal{O}$*-set but not nano open set.*

(2) *the set* $\{1,3,4\}$ *is an ideal nano* $\mathcal{O}$*-set but not an ideal nano* $\mathcal{K}$*-set.*

Proposition 3.6. *Let* P *and* Q *be subsets of a space INTS. If* P *and* Q *are an ideal nano* $\mathcal{K}$*-sets, then* $P \cap Q$ *is an ideal nano* $\mathcal{K}$*-set.*

Proof. Let P and Q be an ideal nano $\mathcal{K}$-sets. Then we have $Nint(P \cap Q) \subset Nint(Ncl^\star(P \cap Q)) \subset Nint(Ncl^\star(P) \cap Ncl^\star(Q) = Nint(Ncl^\star(P)) \cap Nint(Ncl^\star(Q)) = Nint(P) \cap Nint(Q) = Nint(P \cap Q)$.

Then $Nint(P \cap Q) = Nint(Ncl^\star(P \cap Q))$ and hence $P \cap Q$ is an ideal nano $\mathcal{K}$-set.

Example 3.7. *In Example 3.2,*

$P = \{1,3\}$ *and* $Q = \{3,4\}$ *is an ideal nano* $\mathcal{K}$*-set. But* $P \cap Q = \{3\}$ *is an ideal nano* $\mathcal{K}$*-set.*

Proposition 3.8. *For a subset* C *of a space INTS, the next conditions are equivalent:*

(1) C *is nano open,*

(2) C *is ideal nano pre-open and ideal nano* $\mathcal{O}$*-set.*

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Proof. (1) $\Rightarrow$ (2): Let C be nano open. Then $C = Nint(C) \subset Nint(Ncl^{\star}(C))$ and C is an ideal nano pre-open. Also by Remark 3.3 C is an ideal nano $\mathcal{O}$-set.

(2) $\Rightarrow$ (1): Given C is ideal nano $\mathcal{O}$-set. So $C = H \cap K$ where H is nano open and $Nint(K) = Nint(Ncl(K))$. Then $C \subset H = Nint(H)$. Also, C is an ideal anano pre-open implies $C \subset Nint(Ncl(C)) \subset Nint(Ncl^{\star}(K)) = Nint(K)$ by assumption. Thus $C \subset Nint(H) \cap Nint(K) = Nint(H \cap K) = Nint(C)$ and hence C is nano open.

Remark 3.9. *In a space INTS, an ideal nano pre-open sets and an ideal nano $\mathcal{O}$-sets are independent.*

Example 3.10. *In Example 3.2,*

(1) *the set $\{1, 4\}$ is an ideal nano pre-open but not ideal nano $\mathcal{O}$-set.*
(2) *the set $\{2\}$ is an ideal nano $\mathcal{O}$-set but not an ideal nano pre-open.*

Remark 3.11. *In a space ,*

(1) *each an ideal nano $\mathcal{K}_{\alpha}$-set is an ideal nano $\mathcal{O}_{\alpha}$-set.*
(2) *each nano open set is an ideal nano $\mathcal{O}_{\alpha}$-set.*

Example 3.12. *In Example 3.2,*

(1) *the set $\{1, 3, 4\}$ is an ideal nano $\mathcal{O}_{\alpha}$-set but not ideal nano $\mathcal{K}_{\alpha}$-set.*
(2) *the set $\{1\}$ is an ideal nano $\mathcal{O}_{\alpha}$-set but not nano open set.*

Proposition 3.13. *If P and Q are an ideal nano $\mathcal{K}_{\alpha}$-sets of a space INTS, then $P \cap Q$ is a an ideal nano $\mathcal{K}_{\alpha}$-set.*

Proof. Let P and Q be an ideal nano $\mathcal{K}_{\alpha}$-nI-sets. Then we have $Nint(P \cap Q) \subset Nint(Ncl^{\star}(Nint(P \cap Q))) \subset Nint[Ncl^{\star}(Nint(P)) \cap Ncl^{\star}(Nint(Q))]$

$= Nint(Ncl^{\star}(Nint(P))) \cap Nint(Ncl^{\star}(Nint(Q))) = Nint(P) \cap Nint(Q) = Nint(P \cap Q)$. Then $Nint(P \cap Q) = Nint(Ncl^{\star}(Nint(P \cap Q)))$ and hence $P \cap Q$ is an ideal nano $\mathcal{K}_{\alpha}$-nI-set.

Example 3.14. *In Example 3.2,*

the set $\{2,3\}$ and the set $\{1,2\}$ is ideal nano $\mathcal{K}_\alpha$-set. But intersection of both sets for the set $\{2\}$ is an ideal nano $\mathcal{K}_\alpha$-set.

Proposition 3.15. *For a subset C of a space INTS, the next conditions are equivalent:*

(1) *C is nano open.*

(2) *C is an ideal nano α-open and an ideal nano $\mathcal{O}_\alpha$-set.*

Proof. (1) $\Rightarrow$ (2) : Let C be nano open. Then $C = Nint(C) \subset Ncl^\star(Nint(C))$ and $C = Nint(C) \subset Nint(Ncl^\star(Nint(C)))$.

Therefore C is an ideal nano α-open. Also by Remark 3.11(1), C is an ideal nano $\mathcal{O}_\alpha$-set.

(2) $\Rightarrow$ (1) : Given C is an ideal nano $\mathcal{O}_\alpha$-set. So $C = H \cap K$ where H is nano open and $Nint(K) = Nint(Ncl^\star(Nint(K)))$. Then $C \subset H = Nint(H)$. Also C is an ideal nano α-open implies $C \subset Nint(Ncl^\star(Nint(H))) \subset Nint(Ncl^\star(Nint(K))) = Nint(K)$ by assumption.

Thus $C \subset Nint(H) \cap Nint(K) = Nint(H \cap K) = Nint(C)$ and C is nano open.

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